Queueing Theory   Winter 2013
Assignment 1: Probability Overview

1. In answering the multiple choice questions a student either knows the correct answer or she makes a guess. Assume that she knows the answer of each question with probability p and makes a random guess with probability $1 - p$. If the student does not know the answer she can guess the correct answer with probability $\frac{1}{m}$ (m is the number of choices). If the student answers to a question correctly, what is the probability she indeed knew the correct answer and has not made a guess?

2. The screws produced in a factory are defective with probability 0.01. The firm sells the screws in packs where each pack contains ten screws. The firm has guaranteed that at most one of the screws in each pack is defective otherwise the firm takes back the pack and refunds the money. How many percent of the packs should the firm take back?

3. Assume that each engine belonging to a flying airplane can become out of order with probability $1-p$ independent of other engines. In addition, assume that for a successful flight, at least 50% of the engines of the airplane should be working fine. For what value of p a 4 engine airplane is preferred to a 2 engine airplane?

4. Assume that there are 25 different type tokens. Whenever we receive a token it can be one of these 25 types. What is the expected number of token types in a collection of 10 tokens?

5. The probability distribution function for the weekly demand of a product is the following

X	0	1	2
$P(X)$	0.1	0.4	0.5

where X is the weekly demand. Assume that this distribution is identical for all weeks and the demand for each week is independent of other weeks. Find the probability distribution function of the bi-weekly demand.

6. For any random variable $X \geq 0$ show $P\{X \geq a\} \leq \frac{E[X]}{a}$.

7. Consider $n + m$ independent identical trials where each can succeed with probability p. What is the expected number of successes in the first n trials if a total of k successes happen?

8. (Bonus Point) Show that $\lim_{n\to\infty} e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!} = \frac{1}{2}$.