Queueing Theory   Winter 2013
Assignment 2: Probability Overview and Poisson Processes

1. Suppose that the raw materials arrive at a factory based on a Poisson process with parameter λ. These materials all leave the system at the known time T. Find the time $t \in [0, T)$ that if the materials arriving before t all leave the system at this time, then the total expected waiting time of all materials are minimized.

2. Let X be the number of dice throws until the first 6 comes, and Y be the number of dice throws until the first 5. Obtain the following values:

   (a) $\mathbb{E}(X)$

   (b) $\mathbb{E}(X|Y = 1)$

   (c) $\mathbb{E}(X|Y = 3)$

3. Find the function f_n that has the following $z-$transform:

$$P(z) = \frac{1 + 12z}{1 + z + 6z^2}.$$

4. Solve the following difference equation:

$$p_{n+2} - 5p_{n+1} + 6p_n = 0,$$

   where $p_0 = 0$ and $p_1 = 1$.

5. Suppose that X is a random variable from uniform distribution $(0, 50)$ and Y is a random variable from Normal with mean 20 and variance 1. Find the probability that $X > Y$?

6. Consider the independent random variables X_i, $i = 1, 2, ..., n$. Each variable can take the value 1 with probability p and the value 0 with probability $1 - p$. Define a new random variable $S_N = X_1 + X_2 + ... + X_N$ where N is a random variable from Poisson distribution with parameter λ. Show that S_N is random variable from Poisson distribution.

7. Two types of customers arrive at a store. The arrival of each type follows Poisson distributions with parameters 5 and 10 customers per hour respectively.

   (a) What is the probability that the first arrival belongs to a customer of type 1?

   (b) What is the probability that the fifth type 1 customer arrives earlier than the second type 2 customer?

8. Consider independent Bernoulli experiments with the success probability p. What is the average number of trials until we reach to k successive successes?

9. (Bonus) There are n elements $e_1, e_2, ..., e_n$ that are on a list with some order. At each time unit a request arrives related to one of the elements independent of other elements. When a request related to e_i arrives, we place e_i on the top of the list. For example, if the current list is $e_1e_2e_3e_4$ and a request related to e_3 arrives, then the new list becomes $e_3e_1e_2e_4$. Find the average position of a requested element in the long run?