Queueing Theory Winter 2013
Assignment 3: Markov Chains and Exponential Queues

1. A businessman does businesses in the three cities A, B, and C but does not do any business in a city in two successive days. If some day he does a business in city A, the next day he is in B. If some day he does a business in either B or C, the probability that he does a business in A the next day is twice the probability he does a business in the other city.What is the long-run percentage of the time that he spends in each city?

2. There are two white balls (W) in bowl A and three red balls (R) in bowl B. In each trial, two balls are randomly drawn, one from each bowl, and their places are changed. Suppose that a_i represents the state where there are i red balls in bowl A.

   (a) Determine the transition matrix of this Markov chain.

   (b) What is the probability that after three trials there are two red balls in bowl A?

   (c) What is the percentage of the time that bowl A contains two red balls?

3. A textile workshop has 3 old machines and 2 repair operators. The time that each machine works until becomes out of order is exponential with the mean 20 minutes. In addition, the repair time taken for each machine is exponential with the mean 15 minutes.

   (a) What is the probability distribution of the number of machines that are out of order?

   (b) What is the probability distribution of the number of machines that are being repaired?

   (c) What is the percentage of time that both operators are busy?

   (d) If the operators receive a payment only for the time that they work and their salary is \$20 per hour, what is the average payment that the workshop manager should pay per hour?

4. In a queueing system, the service process is exponential with parameter μ and costs $c\mu$ dollars per hour. In addition, the revenue generated from serving each customer is A dollars. If the system's capacity limit is N customers, what is the optima service rate that maximizes the system's profit?

5. Find the values for π_0, π_n, ρ, L and W for a birth-death system with the birth rate $\lambda_n = \frac{\lambda}{n+1}, n = 0, 1, 2, ...$, and the death rate $\mu_n = \mu$, $n = 1, 2, 3, ...$.

6. A barber shop has two hairdressers. The customers arrive at the shop according to a Poisson distribution. If at most one customer is waiting to be served then the customers enter the shop with rate 12 customers per hour. If there are two customers waiting then the arrival rate reduces to 8 customers per hour. If there are 3 customers waiting, then the arrival rate reduces to 4 and finally if there are 4 customers waiting no additional customers are allowed to enter the shop. The service time to each customer is exponential with the mean 10 minutes.

(a) What is the percentage of the time that the shop is empty?

(b) What is the average number of customers in the shop?

(c) What is the average number of customers waiting?

(d) What is the average waiting time in the queue?

(e) What is the probability that an arriving customer is not immediately served?

(f) What is the average number of customers arriving at the shop when a customer is served?

(g) What is the average number of customers entering the shop when a customer is served?