Queueing Theory Winter 2013
Assignment 4

1. Consider a single server queue where arrivals are Poisson with rate $\lambda = 10$ per hour. The service distribution is exponential with rate $\mu = 5$ per hour. Suppose that customers balk at joining the queue when it is too long. Specifically, when there are n customers in the system an arriving customer joins the queue with probability $1/(1+n)$. Determine the stead-state probability that there are n customers in the system.

2. Consider the following model of a single bank teller. Customers arrive according to a Poisson process with rate λ. Customers are served on a FCFS basis. The time to serve each customer is exponential with rate μ. When the teller becomes idle (i.e., when there are no customers in the system), the teller begins a separate off-line task of counting money. The time to complete the money-counting task is exponential with rate γ. Customers who arrive while the teller is counting money must wait until the teller completes the task before receiving any service. If the teller completes the money counting task before any customers arrive, the teller becomes idle until the next customer arrives. The teller then serves the customers until a new idle moment (i.e., when no customers are in the system), at which time the teller again starts a new off-line task of counting money. Suppose the customers are impatient. More specifically, if there are 4 customers in the system, an arriving customer does not join the queue.

   (a) Draw the transition balance diagram for this continuous-time Markov chain and explain why if a sufficient time passes from the start of the system, the probability distribution of the number of customers in the system becomes stationary (i.e., it would not depend on time).

   (b) Find the steady-state probability of the number of customers in the system.

   (c) Find the average number of customers in the system (L) and the average number of customers in the queue (L_q).

3. Consider a work station where its job is to transfer the parts from the first floor to the second floor of a workshop. This station has an elevator with the maximum capacity of transferring 2 parts. The time that it takes to transfer parts from the first to the second floor and turning back to the first floor is exponential with mean μ. The parts arrive at the station according to a Poisson process with parameter λ. Let the state of the system be the number of parts waiting in the queue.

   (a) Draw the transition balance diagram.

   (b) What is the percentage of time that the work station transfers only a single part?

   (c) What is the average number of parts in the queue and in the system (work station)?

4. A firm produces two products, namely Products 1 and 2. The purchase orders that arrive at the firm are first processed in the *marketing* department. In the marketing department, after an initial consideration in section A, each order for Product 1 is sent to section B and for Product 2 to section C. Each order in section B or C goes through

a secondary process and if the inventory is available the order is sent to the *sales* office (section D), otherwise it is sent to the *orders* office where each order goes through an additional administrative process and is then sent to the sales office. The orders arrive at the firm according to a Poisson distribution with mean 8 per hour from which about 70% belong to Product 1. The availability chance of each Product 1 or 2 is 60%. The processing times at the sections A, B, C, and D are exponential with means $6, 7.5, 15$, and 15 minutes, respectively.

(a) What is the average number of orders in the sections A, B, C, and D?

(b) What is the average processing time in the sections A, B, C, and D?

(c) What is the average processing time for an order for Product 1 and for Product 2 in marketing department?

(d) What is the average processing time for an order in marketing department?

(e) What is the probability that there is no order in the marketing department?

(f) What is the probability that there are n orders in the marketing department?

5. Customers arrive at an ATM machine according to a Poisson process with rate $\lambda = 60$ per hour. The following transaction times are observed (in seconds): 28, 71, 70, 70, 51, 62, 36, 25, 35, 87, 69, 27, 56, 25, 36.

   (a) Would an $M/M/1$ queue be an appropriate model for this system (why or why not)?

   (b) Estimate the average number of people waiting in line at the ATM.

   (c) The bank wishes to keep the average line length (number in queue) less than or equal to one. What is the average transaction time needed to achieve this goal (assuming the variance of the transaction time is held constant)?

6. Find the steady-state probabilities for an $M/G/1$ state-dependent queue where

$$B_i(t) = \begin{cases} 1 - e^{-\mu_1 t} & (i = 1) \\ 1 - e^{-\mu t} & (i > 1) \end{cases} .$$

   That is, the service distribution is exponential with mean μ_1 if there is no queue when the customer begins service; the service distribution is exponential with mean μ if there is a queue when the customer begins service.

7. Derive the stationary system-size probabilities for the $M/G/2/2$ queue using only Little's formula and that the fundamental fact that the steady-state inflow and outflow transition rates are equal.